National University Rail Center - NURail
US DOT OST-R Tier 1 University Transportation Center

IMPROVING TRACK SUBSTRUCTURE DESIGNS AND SETTLEMENT DUE TO COMPLEX DYNAMIC LOADS FROM HIGH SPEED PASSENGER AND FREIGHT TRAINS

PROJECT NUMBER: NURAIL2012-UIUC-R03

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INTERIM REPORT

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INTRODUCTION

Railroad track substructures that have traditionally supported heavy freight trains are undergoing rehabilitation as they are developed into shared corridors capable of supporting higher-speed passenger service. Freight train and high-speed passenger train create very different loading patterns for the track system. Freight trains are heavier in axle/wheel loads while slower in speed, which might have a major effect on the resonance frequency of the track system. Hence, track substructures mainly consisting of ballasted track must be strong, durable, and stable enough to withstand repetitive complex dynamic loading without excessive deformation or ride quality degradation. There is an increasing need to (i) better understand the track substructure performance under such demanding dynamic loading scenarios anticipated in shared corridors, (ii) develop new approaches to track substructure designs and rehabilitation methodologies for improved track performance, and hence increased network safety and reliability.

This project aims to improve track substructure designs by properly evaluating effects of mixed-traffic on track response and performance with dynamic analysis of railway track behavior from field instrumentation and analytical modeling. A discretely supported tie, ballast and subgrade track model has been formulated to study such complex dynamic loading patterns, tie-ballast gap and tie support conditions and the related substructure deformations measured under both high speed passenger and heavy freight trains. Associated track settlement, vibration and deterioration trends due to these moving wheel loads can be realistically evaluated using this analytical modeling approach.

This interim report first discusses about a recent field instrumentation study, which involved three North East Corridor (NEC) bridge approach track transition sites of Amtrak passenger lines near Chester, Pennsylvania. The formulation of an analytical model of ballasted track is presented next. Field validation of the analytical model is next accomplished using the measured transient track deformations due to moving wheel loads of a relatively high(er) speed passenger train at the instrumented NEC sites. Finally, field performance of the shared corridor under freight train is predicted using the proposed analytical model.

PREVIOUS MODELING EFFORT

There have been various mathematical and numerical models developed to interpret and predict the dynamic response of railroad track. Early analytical models were one- or two-dimensional ones consisting of a beam as the rail on a Winkler foundation under moving force as wheel load (Mise and Kunii, 1956; Kalker, 1996; Yang et al. 1997; Huang et al. 2009; Basu and Rao, 2013). Some more advanced analytical models also include the mass of train and the interacting force between wheel and track or irregularity of track profile (Zhai and Sun, 1994; Lei and Noda, 2002; Tanabe et al, 2003; Nielson and Oscarsson, 2004; Varandas, 2013). For example, Zhai and Sun (1994) investigated a model for vertical interaction between vehicle and track. Vehicle subsystem was modeled as a multi-body system with 10 degrees of freedom (DOFs) running on the track with a constant velocity, and the track substructure as an infinite Euler beam supported on a discrete continuous elastic foundation consisting of the three layers of rail, crosstie, and ballast.

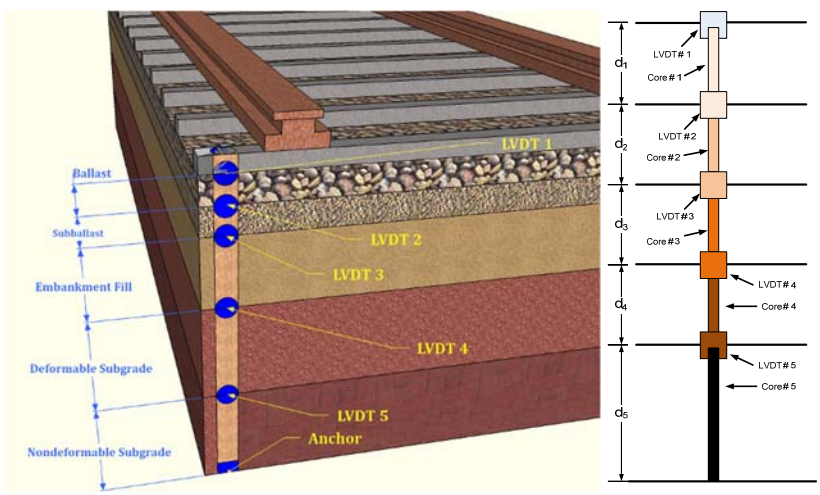
Numerical modeling is another viable alternative for studying the track system under train loading. GEOTRACK (Chang et al. 1980) is a layered elastic system analysis program, which has been validated and widely used for track structural analysis. KENTRACK (Huang et al. 1984) is a finite element-based trackbed structural design program that can be utilized to analyze responses of granular ballast trackbed as well as asphalt trackbed and slab trackbed. However, these models do not consider dynamic response behavior of the track system. To study the time-dependent behavior of track under dynamic train loading, often the Finite Element Method (FEM) has been adopted and used by researchers as the numerical modeling method of choice (Feng, 2011; Smith et al. 2006; Kaewunruen et al. 2011; Giner et al. 2012; Wang et al. 2015).

Among the analytical and numerical models established previously, most of them lacked validation with field data for adequately predicting observed track substructure deformation behavior. Note that validation is essential to check and document whether the model developed is realistic and effective for correctly predicting the field response and track performance. Therefore, there is a need to monitor and numerically model in-service ballasted tracks to predict accurately both transient response and settlement performance of a ballasted track system and accordingly, develop possible mitigation strategies or rehabilitation alternatives to support mixed traffic of freight train and high-speed passenger train.

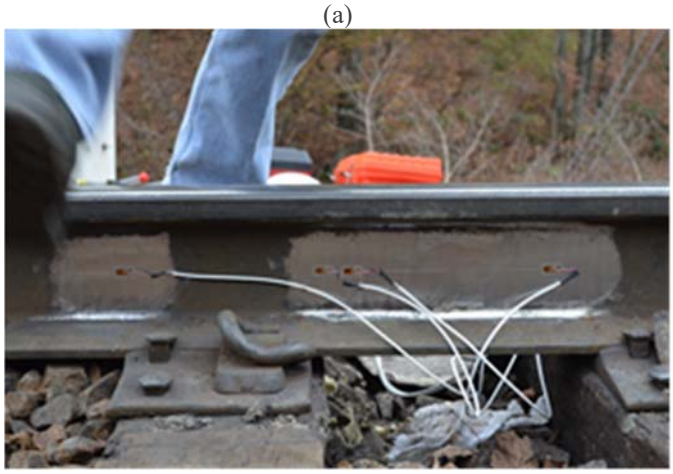
ANALYZING FIELD LOAD-DEFORMATION DATA

Approximately 18 miles south of Philadelphia near Chester, Pennsylvania, a problematic portion of Amtrak's NEC comprises 8 to 10 closely-spaced bridges with recurring differential movement problems at the bridge-embankment interfaces. The NEC is primarily a high-speed railway with occasional freight traffic. The high-speed Acela Express passenger train typically operates at 177 km/h (110 mph) in this location, where three bridge approaches (bridges over Upland, Madison, and Caldwell Streets) with frequent maintenance needs were selected for field instrumentation in a major research project supported by the Federal Railroad Administration (Mishra et al. 2012, Mishra et al. 2017).

Multidepth Deflectometers (MDDs) were selected to monitor the movement of individual track substructure layers. The MDD technology was first developed in South Africa in the early 1980s to measure individual layer deformations in highway pavements (DeBeer et al., 1989). The use of MDDs to monitor railway track performance has been extensively pursued in South Africa (Grabe and Shaw 2010, Priest et al. 2010, Vorster and Grabe 2013) as well as in the U.S. (Sussmann and Selig 1998, Bilow and Li 2005). MDDs consisting of up to six linear variable differential transformers (LVDTs) were installed vertically at preselected depths, which are inside the tie, at bottom of ballast layer, subballast layer, embankment fill layer, deformable subgrade layer, and nondeformable subgrade to measure the displacements of individual substructure layers. Beside the MDDs, strain gauges were also mounted on the rail to measure vertical wheel loads applied during the passage of a train. Dual-element 350-Ohm shear gauges were welded on the rail at the neutral axis both on top of the tie and in the middle point of two adjacent ties to measure tie reaction force as well as vertical wheel loads. Figure 1(a) illustrates the location of each MDD module placed at layer interfaces in the track substructure. Figure 1(b) illustrates the location of the strain gauge on the rail. More details of the field instrumentation can be found in Mishra et al. (2012).



(a)



(b)

Figure 1. Field Installations of (a) Multidepth Deflectometers (MDDs) and (b) Strain Gauges

In this track transition field study, bridge approach transient subsurface layer deformations as well as corresponding vertical wheel loads were measured at the instrumented crosstie locations. Figure 2 shows an example of vertical wheel loads applied on the rail under a passing ACELA passenger train. The 32 peaks correspond to the 32 wheels in an Acela Express passenger train operating along the instrumented location. Two locomotives (one at each end of the train) and six passenger cars are registered. It can be seen that locomotives generate higher loads (approximately 140 kN) compared to the passenger cars (approximately 90 kN). Note that the measured loads consisted of both the static weight of the car and the dynamic load caused by any impact loading as well as track irregularities and defects.

Figure 3 shows the transient deformation time history recorded by individual LVDT modules under the same passing high-speed Acela Express passenger train. Note that the peak deformations corresponding to the passage of each wheel are quite clear. Furthermore, LVDT 1 mounted inside the crosstie indicates the highest transient deformation recorded among all the LVDTs installed, each measuring individual layer deformations. However, the deformation of subballast layer registered by LVDT 2 is in fact lower than that of embankment and subgrade layers.

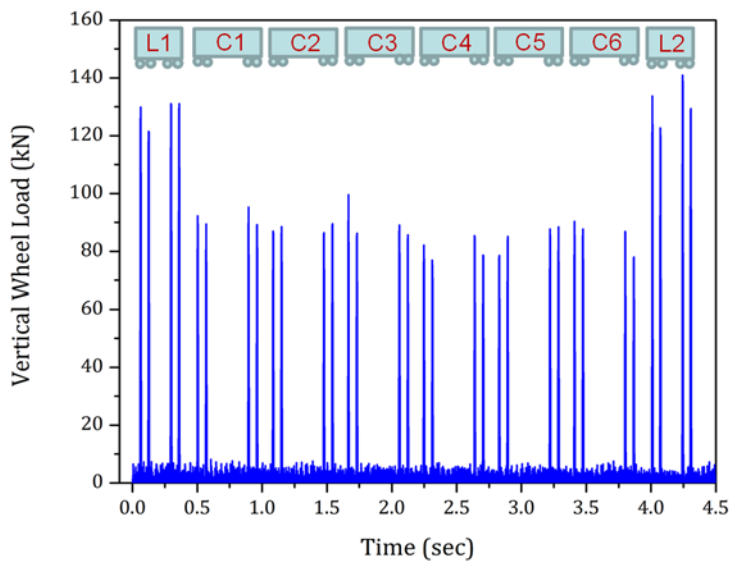


Figure 2. Field Recorded Load Time History of Acela Express Passenger Train

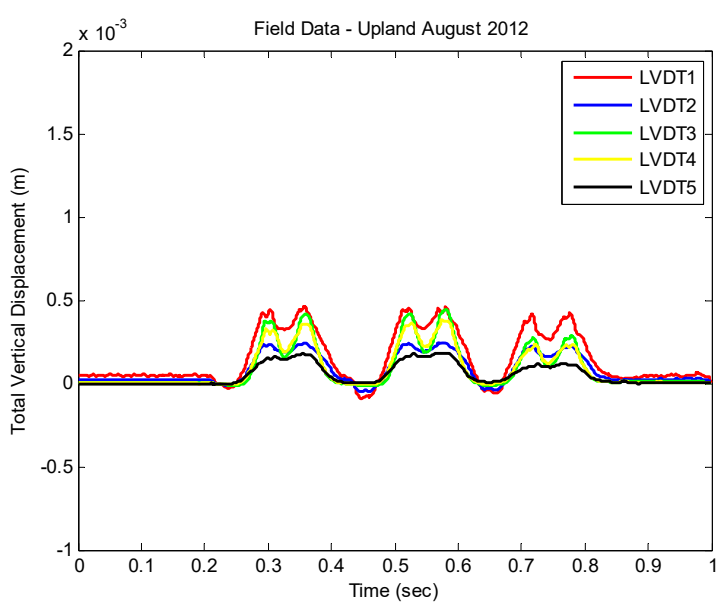


Figure 3. Field Recorded Transient Deformation Time History at Upland Street

Analytical Modeling of Track under High-Speed Train Loading

In this section, an analytical model developed with the assumption of discretely supported crossties in-service on both the approach embankment and bridge deck under moving wheel loads is introduced. Figure 4 illustrates the various components of the ballasted track model. The locomotive is assumed to travel in the direction of arrow at a constant speed of v. Track substructure on the embankment side is modeled as an infinite Euler beam discretely supported by three layers of viscous-elastic foundations consisting of crosstie, ballast, and subgrade soil. Track substructure on the bridge side is also modeled as an infinite Euler beam discretely supported this time by one layer of viscous-elastic foundation representing crossties resting directly on top of Winkler foundation. It is important to note that the model can analyze track substructure behavior with either one side (bridge approach embankment or bridge deck) or both sides considered to conduct analysis. For example, to focus only on open track analysis of the ballasted track system, the bridge side can be neglected by assigning no ties on bridge deck. This way, the model is available to analyze an open track system. When both the open track and bridge deck sides are included, moving wheel loads of a high-speed train, e.g., entering ballasted bridge deck, will be possible to analyze to gain a better understanding of dynamic load effects on substructure deformation behavior.

Since the analytical model developed only considers the vertical direction of train deformations, equations of motion of the track system can be easily derived as follows:

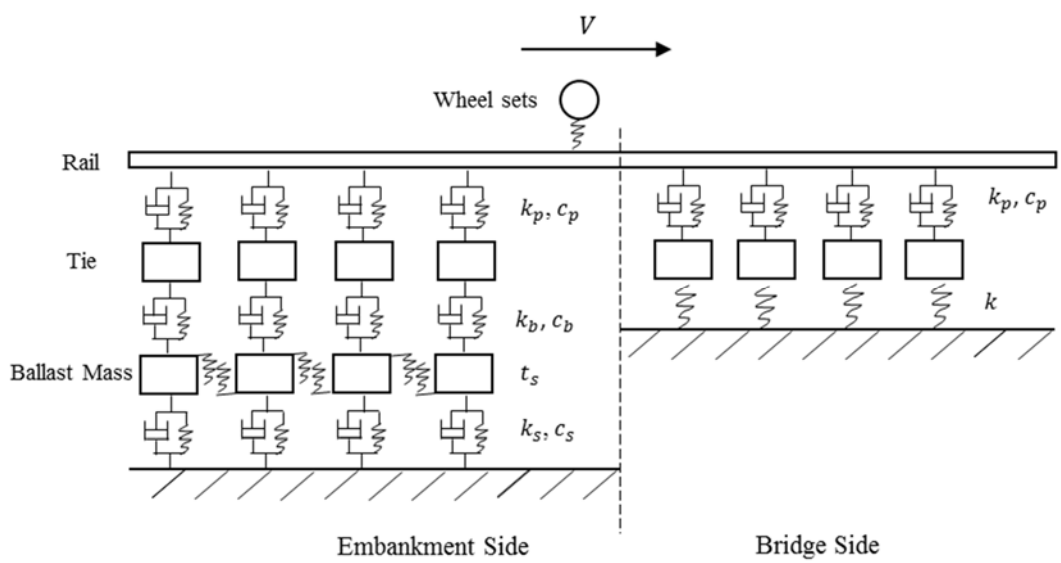


Figure 4. Analytical Track Model Developed

Embankment side:

Equation of motion of rail beam is given by

$$EI\frac{\partial^4 U_r(x,t)}{\partial x^4} + \rho_r \frac{\partial^2 U_r(x,t)}{\partial t^2} + \sum_1^M a_m(t)\delta(x - x_m) = F\delta(x - vt) \quad \text{(F is moving load)}$$

Equation of motion of rail-tie reaction is given by

$$a_m(t) = c_p\left(\frac{\partial U_r(x_m,t)}{\partial t} - \frac{\partial U_t(x_m,t)}{\partial t}\right) + k_p(U_r(x_m,t) - U_t(x_m,t))$$

Equation of motion of tie is given by

$$m_t\frac{\partial^2 U_t(x_m,t)}{\partial t^2} = a_m(t) - b_m(t)$$

Equation of motion of tie-ballast reaction is given by

$$b_m(t) = c_b\left(\frac{\partial U_t(x_m,t)}{\partial t} - \frac{\partial U_b(x_m,t)}{\partial t}\right) + k_b(U_t(x_m,t) - U_b(x_m,t))$$

Equation of motion of ballast mass is given by

$$m_b \frac{\partial^2 U_b(x_m,t)}{\partial t^2} = b_m(t) - c_s \frac{\partial U_b(x_m,t)}{\partial t} - k_s U_b(x_m,t) + T_m - T_{m+1}$$

Equation of motion of the shear force between ballast mass is given by

$$T_m = t_s(U_b(x_{m-1},t) - U_b(x_m,t))$$

where

U_r, U_t, U_b are unknown variables, representing deflection of rail, tie, and the ballast mass, respectively;
k_p, k_b, k_s are stiffness properties of rail pad, ballast, and the subgrade layer;
c_p, c_b, c_s are damping ratios of rail pad, ballast, and the subgrade layer;
EI is bending stiffness of the rail; and
t_s is shear stiffness between ballast masses.

Following the work by Kalker (1996) and Huang et al. (2009), Fourier Transform technique can be utilized to solve for the equations of motion of the system. The Fourier Transform was first performed from time to frequency domain and from spatial domain to wave length domain.

$$(EI\lambda^4 - \omega^2\rho)U_r(\lambda,\ \omega) = F^t\left(\frac{Fe^{-\frac{ix\omega}{v}}}{v} - \sum\nolimits_N a_m(\omega)\delta(x - x_m)\right)$$

$$a_m(\omega) = \left(c_p i\omega + k_p\right)\left(U_r(x_m,\omega) - U_t(x_m,\omega)\right)$$

$$b_m(\omega) = (c_b i\omega + k_b)\left(U_t(x_m,\omega) - U_b(x_m,\omega)\right)$$

$$-m_t\omega^2 U_t(x_m,\omega) = a_m(\omega) - b_m(\omega)$$

$$(-m_b\omega^2 + c_s i\omega + k_s + 2t_s)U_b(x_m,\omega) = b_m(\omega) + t_s U_b(x_{m-1},\omega) + t_s U_b(x_{m+1},\omega)$$

where

F^t means Fourier transform from spatial x to wave length.

The inverse Fourier Transform was next performed on equation of rail beam from wave length domain back to spatial domain. The equation of motion for rail beam then becomes:

$$U_r(x,\ \omega) = -\frac{F\sqrt{R}}{4EIR^2 v}\int_0^{vT}\left(ie^{-\frac{iy\omega}{v} - i|x-y|\sqrt{R}} + e^{-\frac{iy\omega}{v} - |x-y|\sqrt{R}}\right)dy\ - \sum\nolimits_N a_m(\omega)K_r(x - x_m)$$

where

$$K_r(x,\omega) = -\frac{1}{4EIR^2}\left(\sqrt{R}e^{-|x|\sqrt{R}} + i\sqrt{R}e^{-i|x|\sqrt{R}}\right)$$

Accordingly, all the equations are assembled into a matrix form (including both embankment side and bridge side):

$$\begin{bmatrix} BM & 0 \\ AK & +I \end{bmatrix}_{(M+N)\times N}[U_r]_{N\times 1} + \begin{bmatrix} Dt_s \\ -BK \end{bmatrix}_{(M+N)\times M}[U_b]_{M\times 1} = \begin{bmatrix} 0 \\ FG \end{bmatrix}_{(M+N)\times 1}$$

where

M is the number of ties on embankment side;
$M - N$ is the number of ties on bridge side (will be assigned as zero for analysis in this report); and

$$Dt_s = \begin{bmatrix} D_1 & t_s & 0 & & 0 & 0 & 0 \\ -t_s & D_2 & t_s & \cdots & 0 & 0 & 0 \\ 0 & -t_s & D_3 & & 0 & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ 0 & 0 & 0 & & -t_s & D_{M-1} & t_s \\ 0 & 0 & 0 & \cdots & 0 & -t_s & D_M \end{bmatrix}$$

$$BM = \begin{bmatrix} -B_1 & 0 & 0 & & & \\ 0 & -B_2 & 0 & \cdots & & 0 \\ 0 & 0 & -B_3 & & & \\ & \vdots & & \ddots & & \vdots \\ & & & & -B_{M-1} & 0 \\ & 0 & & \cdots & 0 & -B_M \end{bmatrix}$$

$$FG(x,\omega) = -\frac{F\sqrt{R}}{4EIR^2 v}\int_0^{vT}\left(ie^{-\frac{iy\omega}{v}-i|x-y|\sqrt{R}} + e^{-\frac{iy\omega}{v}-|x-y|\sqrt{R}}\right)dy$$

$$AK(n,m) = A_m(m)K_r(x_n - x_m)$$
$$BK(n,m) = B_m(m)K_r(x_n - x_m)$$
$$A_m(open\ track\ side) = \frac{(-m_t\omega^2 + CK_b)\cdot CK_p}{-m_t\omega^2 + CK_p + CK_b}$$
$$B_m = \frac{CK_b \cdot CK_p}{-m_t\omega^2 + CK_p + CK_b}$$
$$C_m = \frac{(-m_t\omega^2 + CK_p)\cdot CK_b}{-m_t\omega^2 + CK_p + CK_b}$$
$$D_m = -m_b\omega^2 + c_s i\omega + k_s + 2t_s + C_m$$
$$CK_p = (c_p i\omega + k_p)$$
$$CK_b = (c_b i\omega + k_b)$$

Open track in the Upland Street bridge approach location is used here as a validation case to compare its measured transient deformations under Acela Express train loading to the predictions obtained from the analytical model. The parameters used in the model are listed in Table 1. With the chosen parameters and derived matrix for solving the mathematical equations of motion, a computer program based on MATLAB was developed. In this case study, the track model included 30 ties spaced at 2 ft. (609.6 mm) on the embankment side only.

Table 1. Track Parameters Used in the Model

Example Case: Upland 60 ft., August 2012	
Track variable	Value
(1) Rail Properties	
E (MPa)	2.07E+05
Weight (kg/m)	67.46
I (mm4)	3.90E+07
Crosstie pad stiffness (kN/m)	1.20E+05
Crosstie pad damping (kN.s/m)	124
(2) Crosstie Properties	
Center to center sleeper spacing (mm)	609.6
Sleeper weight (kg)	386
(3) Ballast Properties	
Ballast mass (kg)	683
Ballast stiffness (kN/m)	7.0E+04
Ballast damping (kN.s/m)	82
Ballast shear stiffness (kN/m)	7.8E+03
(4) Subgrade Properties	
Subgrade stiffness (kN/m)	6.5E+04
Subgrade damping (kN.s/m)	30

To better illustrate the effect of multiple-wheel-passage, six bogies of wheel sets were applied in the model using superposition. The first four loads were for the locomotive of Acela Express (each one is approximately 140,000 N) and two consecutive loads for the passenger car of Acela Express (each one is approximately 90,000 N) in accordance with the strain gauge measurements at the Upland Street bridge approach site. The bogies were separated from each other at distances of 2800 mm, 7900 mm, 2800 mm, 6500 mm, and 2800 mm, respectively from the first wheel to the sixth wheel loading. The wheel loads were moving at a constant speed of 50 m/s (110 mph) in accordance with Acela Express train speed observed in the field.

Figure 5 shows the response of the rail beam (represents the total vertical transient deformation) in the model under six moving loads on the embankment side. Note that the rail first experiences an uplifting behavior before the first wheel load moves to top. It then oscillates when the first moving load leaves. Due to superposition, which does not account for interaction between wheels, the oscillation behavior between adjacent wheels may be magnified. The maximum amount of downward deflection of rail predicted by track transition model is approximately 1.7 mm. The maximum amount of upward deflection predicted by track transition model is approximately 0.4 mm. After the passage of the passenger train, the rail deflection quickly goes down to almost zero, which is mainly due to the high natural frequency of rail.

To evaluate the prediction accuracy of the analytical model, the deflections predicted by the model were compared to the field collected displacement data recorded by the installed MDDs. Figure 6 shows the field transient deformation time history data for Upland Street (only the data corresponding to the locomotive and first passenger car wheel loads are graphed). The field deformations recorded by LVDT 1 through LVDT 5 are summed to account for the total vertical deformation. Note that the total vertical displacements and the transient response trends predicted by the model are similar in magnitude to those measured in the field. The displacements may be shifted in time when comparing field data to the model results due to the difference of start time of load application. The maximum displacement under moving wheel load of the locomotive is around 1.7 mm measured by MDDs in the field. Both the field data and the model predictions indicate the potential of uplifting before the first wheel load and between two adjacent wheel loads. However, it should be noted that the results from the model can overestimate the heaving behavior of track system under loading. This is probably due to the tension provided by springs (assumption in the model) which can pull the component back to normal position while in reality no such tension occurs in an unbound ballast layer or between unconnected components such as between tie and ballast. Though the magnitudes of deflection predicted by the model are close to field measurements, it is obvious that the oscillation registered by the model is more significant. This may be contributed by a series of springs employed in the model.

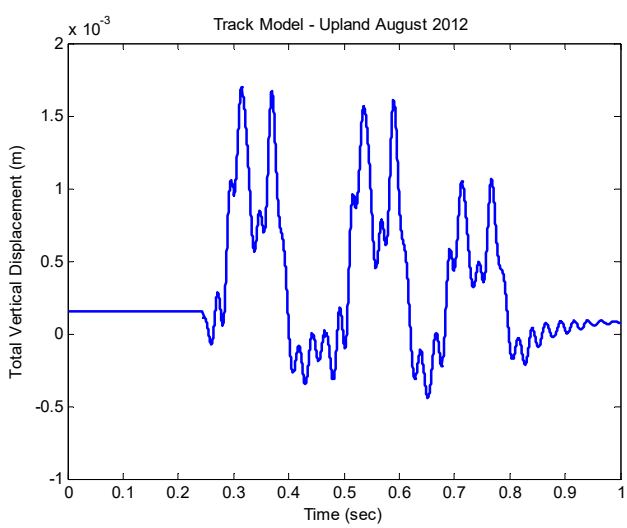


Figure 5. Total Vertical Transient Deformations Predicted by Analytical Model under High-speed Passenger Train

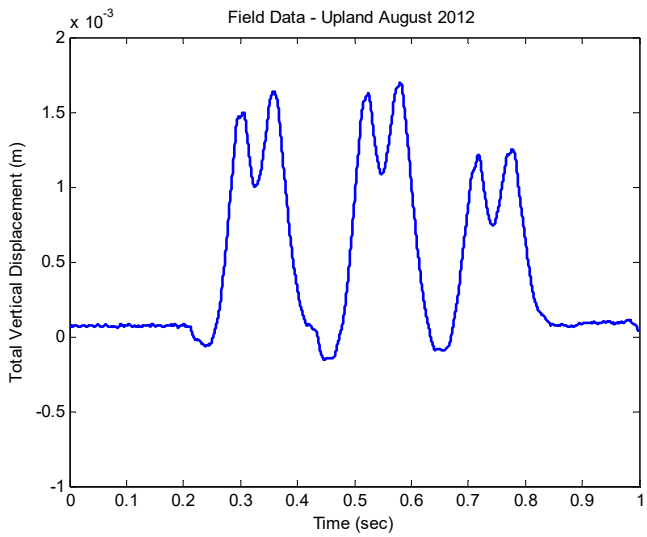


Figure 6. Total Vertical Transient Deformations Measured in the Field under High-speed Passenger Train

Evaluation of Track under Freight Rail Loads

Shared corridors support both higher-speed passenger service and freight trains. As stated earlier, the instrumented NEC is primarily a high-speed railway with occasional freight traffic, which qualifies as a shared corridor. The analytical model developed for ballasted track has been validated with field data recorded under the passage of high-speed Acela Express passenger train, which operates at 177 km/h (110 mph) in Chester, PA. This section presents an application of the analytical model for evaluating the track deformation behavior under freight train loading.

All the track substructure parameters (stiffness and damping ratios of various components) were assumed the same as in the previous field validation model (see Table 1). The track model also included 30 ties spaced at 2 ft. (609.6 mm) on the embankment side only. To simulate the moving loads of freight train, six moving wheel loads were applied with each load assumed to be 150,000 N. The bogies were separated from each other at the same distances as Acela Express for better comparison, which are 2800 mm, 7900 mm, 2800 mm, 6500 mm, and 2800 mm from the first wheel to the sixth wheel loading, respectively. The wheel loads were moving at a constant speed of 11 m/s (25 mph) similar to the maximum speed of the freight train observed at night in the instrumented location.

Figure 7 shows the total vertical deformation time history predicted by the analytical model. Please note that to compare freight train and high-speed passenger train loadings, a duration of 1-second was chosen as total observation time. Note that only two peaks are registered in Figure 7, which is due to the slow speed of freight train. During the 1-second observation time, only two wheel loads pass over the MDD instrumented crosstie. As stated earlier, the oscillation predicted by the analytical model is due to springs and dampers employed. Hence, the magnitude of maximum deformation and the general deformation trends are appropriate results one should consider when comparing track system performances under the freight train and high-speed passenger train. Figure 7 shows that the maximum transient deformation caused by the freight train is lower than that caused by high-speed passenger train even though the axle load by freight train is higher. Note that speed of the operating train is a major component that contributes to the track system performance. Whereas, maximum uplifting deformation caused by freight train is significantly higher than that caused by high-speed passenger train. It is attributed to the heavier weight of freight train (static loading) that causes the upward movement of rail when the load is not directly on top of the observation point. As far as the total deformation (distance between maximum downward deformation and maximum upward deformation) is considered, the freight train prediction of approximately 2.4 mm can be more critical than the 2.1 mm high-speed passenger train value. The deformation trends of freight train and high-speed passenger train also differ with each other. Figure 5 indicates sharp peaks predicted for the transient deformation under high-speed passenger train under each axle loading, whereas, peaks of transient deformation under freight train are more widely distributed (see Figure 7).

No doubt, more ballasted track system analyses and model prediction results, as part of the ongoing research efforts, are needed to adequately establish such different track deformation trends under complex loading conditions by both freight train and high-speed train.

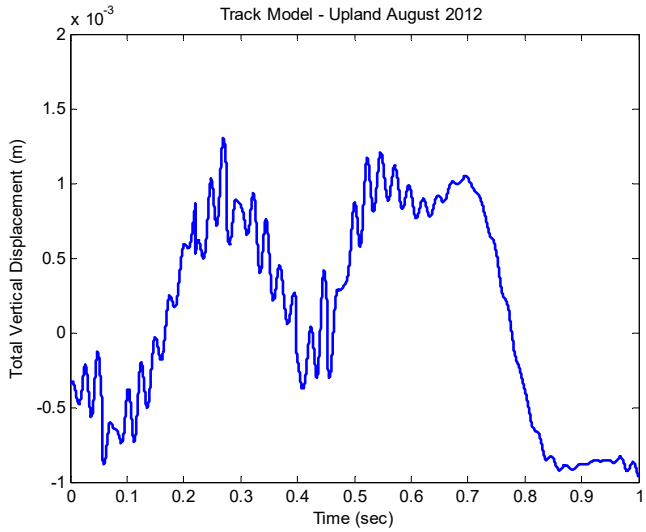


Figure 7. Total Vertical Transient Deformations Predicted by Analytical Model under Freight Train

SUMMARY, CONCLUSIONS, RECOMMENDATIONS

This report provided summary results of the ongoing research at the University of Illinois at Urbana-Champaign, supported by project NURail2013-UIUC-R10. A discretely supported track model was introduced to consider both bridge approach embankment and bridge deck sides in the analysis of substructure deformation behavior. The analytical model is able to simulate track system under one or multiple constant moving wheel loads using superposition. The model formulation presented was programmed in a computer program to conduct analysis and evaluate the performance of any track system. Field instrumentation and recorded time history data from the Northeast Corridor (NEC) Amtrak passenger and freight rail lines were presented next. One case study was conducted for one of the field instrumented of bridge approach sites on Amtrak's NEC location (Upland Street location near Chester, Pennsylvania), where the high-speed Acela Express passenger train operates at 50m/s (110 mph). The transient open track rail displacements predicted from the model adequately matched the field measured data for the maximum displacement and the transient response trends under loading. With some model validation accomplished, the deformation behavior of the same track substructure this time under a freight train loading was adequately predicted using the developed model at the same field site. The freight train was assumed to operate at 11m/s (25mph). Interestingly, the track vertical deformations predicted due to the freight train and the high-speed passenger train loadings, realistically assessed from wheel load measurements in the field, were quite different. The passenger train could cause higher deformations with sharp peaks at the axle load positions. Whereas, the freight train could result in cause higher bounce-back type upward deformations with more widely distributed "peaks" corresponding to axle loads.

Future work will further develop and validate the analytical track model using a wide range of field collected data from two other instrumented sites at Amtrak's NEC. Wheel-rail interaction will be studied to incorporate related features in the track model so that interaction behavior of excitations and vibrations of vehicle and rail systems can be considered. The model will consider different movements of the substructure, for example, deformations predicted not only in the vertical but also in the lateral direction. Once the model is fully developed and validated with a complete set of available field instrumentation data (including bridge approach differential movements), it can be used to study field performance trends under different train speeds, train weights, and for example, effects of hanging tie. Further, some of the remedial measures commonly applied to ballasted track, for example, under tie pads or installed ballast mats on bridge deck, can be studied using the validated model to better understand their working principles as well as to assess their effectiveness for mitigating effects caused by complex loading regimes of freight and high-speed passenger trains.

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